Valori delle funzioni trigonometriche di angoli particolari

	sen	cos	tg	ctg
15°	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
18°	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{5+2\sqrt{5}}$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
36°	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{5-2\sqrt{5}}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1
54°	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$	$\sqrt{5-2\sqrt{5}}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$
72°	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$
75°	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$

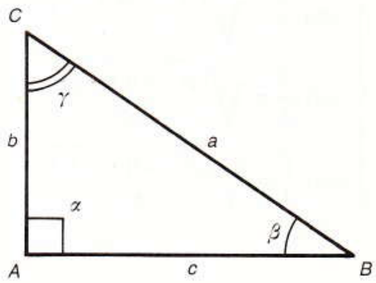
Schema riassuntivo delle formule presenti nel testo

Sui triangoli rettangoli

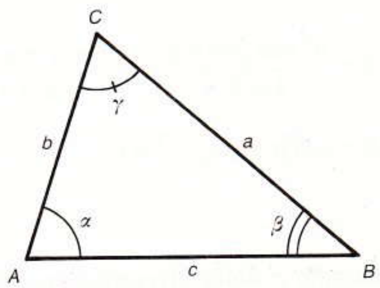
$$\frac{b}{a}=\operatorname{sen}\beta=\cos\gamma$$

$$\frac{c}{a}=\cos\beta=\operatorname{sen}\gamma$$

$$\frac{b}{c}=\operatorname{tg}\beta=\operatorname{ctg}\gamma$$



Sui triangoli qualunque



Teorema dei seni $\frac{a}{\operatorname{sen}\alpha}=\frac{b}{\operatorname{sen}\beta}=\frac{c}{\operatorname{sen}\gamma}$

Teorema del coseno (o di Carnot)

$$\begin{cases} a^2=b^2+c^2-2bc\cos\alpha \\ b^2=a^2+c^2-2ac\cos\beta \\ c^2=a^2+b^2-2ab\cos\gamma \end{cases}$$

Teorema delle proiezioni

$$\begin{cases} a=b\cos\gamma+c\cos\beta \\ b=c\cos\alpha+a\cos\gamma \\ c=a\cos\beta+b\cos\alpha \end{cases}$$

Teorema di Nepero (delle tangenti)

$$\begin{cases} \operatorname{tg}\frac{\alpha-\beta}{2}=\frac{a-b}{a+b}\operatorname{ctg}\frac{\gamma}{2} \\ \operatorname{tg}\frac{\beta-\gamma}{2}=\frac{b-c}{b+c}\operatorname{ctg}\frac{\alpha}{2} \\ \operatorname{tg}\frac{\gamma-\alpha}{2}=\frac{c-a}{c+a}\operatorname{ctg}\frac{\gamma}{2} \end{cases}$$

Formule di Delambre

$$\begin{cases} \dfrac{a+b}{c} = \dfrac{\cos \dfrac{\alpha-\beta}{2}}{\cos \dfrac{\alpha+\beta}{2}} \\[2ex] \dfrac{a-b}{c} = \dfrac{\operatorname{sen} \dfrac{\alpha-\beta}{2}}{\operatorname{sen} \dfrac{\alpha+\beta}{2}} \end{cases} \qquad \text{e analoghe}$$

Formule di Briggs

$$\begin{cases} \operatorname{sen}\dfrac{\alpha}{2} = \sqrt{\dfrac{(p-b)(p-c)}{bc}} \\ \operatorname{sen}\dfrac{\beta}{2} = \sqrt{\dfrac{(p-c)(p-a)}{ac}} \\ \operatorname{sen}\dfrac{\gamma}{2} = \sqrt{\dfrac{(p-a)(p-b)}{ab}} \end{cases} \qquad \begin{cases} \cos\dfrac{\alpha}{2} = \sqrt{\dfrac{p(p-a)}{bc}} \\ \cos\dfrac{\beta}{2} = \sqrt{\dfrac{p(p-b)}{ac}} \\ \cos\dfrac{\gamma}{2} = \sqrt{\dfrac{p(p-c)}{ab}} \end{cases} \qquad \begin{cases} \operatorname{tg}\dfrac{\alpha}{2} = \sqrt{\dfrac{(p-b)(p-c)}{p(p-a)}} \\ \operatorname{tg}\dfrac{\gamma}{2} = \sqrt{\dfrac{(p-c)(p-a)}{p(p-b)}} \\ \operatorname{tg}\dfrac{\gamma}{2} = \sqrt{\dfrac{(p-a)(p-b)}{p(p-c)}} \end{cases}$$

con $2p = a+b+c$.

Area S

$$S = \frac{1}{2} ab \operatorname{sen} \gamma = \frac{1}{2} bc \operatorname{sen} \alpha = \frac{1}{2} ac \operatorname{sen} \beta$$

$$S = \frac{a^2 \operatorname{sen} \beta \operatorname{sen} \gamma}{2 \operatorname{sen} \alpha} = \frac{b^2 \operatorname{sen} \alpha \operatorname{sen} \gamma}{2 \operatorname{sen} \beta} = \frac{c^2 \operatorname{sen} \alpha \operatorname{sen} \beta}{2 \operatorname{sen} \gamma}$$

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$ **(Formula di Erone)**

Raggio r della circonferenza inscritta

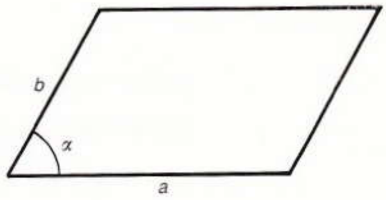
$$r = \frac{S}{p} = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

Raggio R della circonferenza circoscritta

$$R = \frac{a}{2 \operatorname{sen} \alpha} = \frac{b}{2 \operatorname{sen} \beta} = \frac{c}{2 \operatorname{sen} \gamma}$$

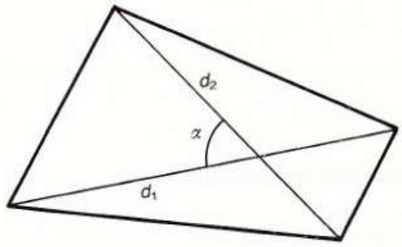
$$R = \frac{abc}{4S} = \frac{abc}{4\sqrt{p(p-a)(p-b)(p-c)}}$$

Sui quadrilateri



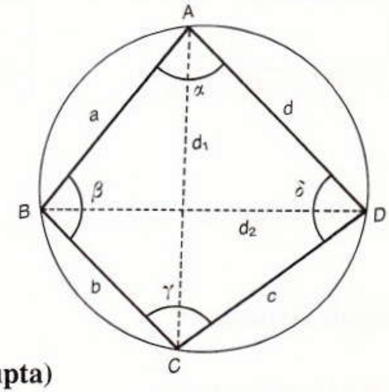
Area S del parallelogramma

$S=ab \operatorname{sen} \alpha$



Area S del quadrilatero

$S=\frac{1}{2} d_1 \cdot d_2 \operatorname{sen} \alpha$



Sul quadrilatero inscritto in un cerchio

Teorema di Legendre $\frac{d_1}{d_2}=\frac{bc+ad}{cd+ab}$

Teorema di Tolomeo $d_1 d_2=ac+bd$

Area $S=\frac{1}{2}(ad+bc) \operatorname{sen} \alpha$

$S=\sqrt{(p-a)(p-b)(p-c)(p-d)}$ **(Formula di Brahamagupta)**

(con $2p=a+b+c+d$).

Se il quadrilatero è anche circoscritto ad un cerchio

$S=\sqrt{abcd}$

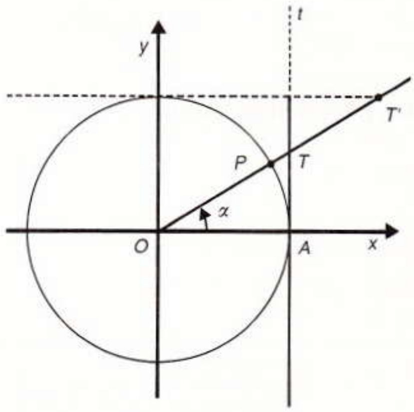
Sulle funzioni circolari

$\operatorname{sen}\alpha =$ ordinata di P

$\cos\alpha =$ ascissa di P

$\operatorname{tg}\alpha =$ ordinata di T

$\operatorname{ctg}\alpha =$ ascissa di T'



Relazioni fondamentali

$$\operatorname{sen}^2\alpha+\cos^2\alpha=1$$

$$\frac{\operatorname{sen}\alpha}{\cos\alpha}=\operatorname{tg}\alpha$$

$$\operatorname{ctg}\alpha=\frac{1}{\operatorname{tg}\alpha} \qquad \sec\alpha=\frac{1}{\cos\alpha} \qquad \operatorname{cosec}\alpha=\frac{1}{\operatorname{sen}\alpha}$$

$$\begin{cases}\cos\alpha=\pm\sqrt{1-\operatorname{sen}^2\alpha}\\ \operatorname{tg}\alpha=\dfrac{\operatorname{sen}\alpha}{\pm\sqrt{1-\operatorname{sen}^2\alpha}}\end{cases} \qquad \begin{cases}\operatorname{sen}\alpha=\pm\sqrt{1-\cos^2\alpha}\\ \operatorname{tg}\alpha=\dfrac{\pm\sqrt{1-\cos^2\alpha}}{\cos\alpha}\end{cases} \qquad \begin{cases}\operatorname{sen}\alpha=\dfrac{\operatorname{tg}\alpha}{\pm\sqrt{1+\operatorname{tg}^2\alpha}}\\ \cos\alpha=\dfrac{1}{\pm\sqrt{1+\operatorname{tg}^2\alpha}}\end{cases}$$

Angoli associati

$$\begin{cases}\operatorname{sen}(-\alpha)=-\operatorname{sen}\alpha\\ \cos(-\alpha)=\cos\alpha\\ \operatorname{tg}(-\alpha)=-\operatorname{tg}\alpha\end{cases} \qquad \begin{cases}\operatorname{sen}(90°-\alpha)=\cos\alpha\\ \cos(90°-\alpha)=\operatorname{sen}\alpha\\ \operatorname{tg}(90°-\alpha)=\operatorname{ctg}\alpha\end{cases} \qquad \begin{cases}\operatorname{sen}(90°+\alpha)=\cos\alpha\\ \cos(90°+\alpha)=-\operatorname{sen}\alpha\\ \operatorname{tg}(90°+\alpha)=-\operatorname{ctg}\alpha\end{cases}$$

$$\begin{cases}\operatorname{sen}(180°-\alpha)=\operatorname{sen}\alpha\\ \cos(180°-\alpha)=-\cos\alpha\\ \operatorname{tg}(180°-\alpha)=-\operatorname{tg}\alpha\end{cases} \qquad \begin{cases}\operatorname{sen}(180°+\alpha)=-\operatorname{sen}\alpha\\ \cos(180°+\alpha)=-\cos\alpha\\ \operatorname{tg}(180°+\alpha)=\operatorname{tg}\alpha\end{cases} \qquad \begin{cases}\operatorname{sen}(360°-\alpha)=-\operatorname{sen}\alpha\\ \cos(360°-\alpha)=\cos\alpha\\ \operatorname{tg}(360°-\alpha)=-\operatorname{tg}\alpha\end{cases}$$

Formule di addizione e sottrazione

$\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha \cos\beta \pm \cos\alpha \cos\beta$

$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \operatorname{sen}\alpha \operatorname{sen}\beta$

$\operatorname{tg}(\alpha + \beta) = \dfrac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha \operatorname{tg}\beta}$

Formule di duplicazione

$\operatorname{sen} 2\alpha = 2 \operatorname{sen}\alpha \cos\alpha$ $\qquad \cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha = \begin{cases} 1 - \operatorname{sen}^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\qquad \operatorname{tg} 2\alpha = \dfrac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$

Formule di triplicazione

$\operatorname{sen} 3\alpha = 3\operatorname{sen}\alpha - 4\operatorname{sen}^3\alpha$ $\qquad \cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$ $\qquad \operatorname{tg} 3\alpha = \dfrac{3\operatorname{tg}\alpha - \operatorname{tg}^3\alpha}{1 - 3\operatorname{tg}^2\alpha}$

Formule di bisezione

$\operatorname{sen}\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$ $\qquad \cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$ $\qquad \operatorname{tg}\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}} = \dfrac{\operatorname{sen}\alpha}{1+\cos\alpha}$

Formule parametriche

$\operatorname{sen}\alpha = \dfrac{2t}{1+t^2}$ $\qquad \cos\alpha = \dfrac{1-t^2}{1+t^2}$ $\qquad \operatorname{tg}\alpha = \dfrac{2t}{1-t^2}$ $\qquad \left(\text{con } t = \operatorname{tg}\dfrac{\alpha}{2}\right)$

Formule di Werner

$\operatorname{sen}\alpha \operatorname{sen}\beta = \dfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\cos\alpha \cos\beta = \dfrac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

$\operatorname{sen}\alpha \cos\beta = \dfrac{1}{2}[\operatorname{sen}(\alpha+\beta) + \operatorname{sen}(\alpha-\beta)]$

$\cos\alpha \operatorname{sen}\beta = \dfrac{1}{2}[\operatorname{sen}(\alpha+\beta) - \operatorname{sen}(\alpha-\beta)]$

Formule di prostaferesi

$\operatorname{sen} p + \operatorname{sen} q = 2\operatorname{sen}\left(\dfrac{p+q}{2}\right)\cos\left(\dfrac{p-q}{2}\right)$

$\operatorname{sen} p - \operatorname{sen} q = 2\cos\left(\dfrac{p+q}{2}\right)\operatorname{sen}\left(\dfrac{p-q}{2}\right)$

$\cos p + \cos q = 2\cos\left(\dfrac{p+q}{2}\right)\cos\left(\dfrac{p-q}{2}\right)$

$\cos p - \cos q = -2\operatorname{sen}\left(\dfrac{p+q}{2}\right)\operatorname{sen}\left(\dfrac{p-q}{2}\right)$